Roll No. ....

Y – 3180 (A)

M.A./M.Sc. (Mathematics) (Fourth Semester) (SPECIAL)

EXAMINATION, August 2021

(SECOND CHANCE)

Paper – 401

PARTIAL DIFFERENTIAL EQUATION

Time : Three Hours

Maximum Marks : 85 *Minimum Pass Marks : 29*

Note—Attempt *all* questions.

1. Find the integral surface of the linear partial differential equation— 17

$$x(y^2 + z)\, p - y\,(x^2 + z)q = (x^2 - y^2)\, z$$

2. Reduce the following equation to a canonical form— 17

$$(1+x^2)\, U_{xx} + (1 + y^2)\, U_{yy} + xu_x + yu_y = 0$$

3. Show that in cylindrical coordinates r, θ, z defined by the relation $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, the Laplace equation $\nabla^2 u = 0$ takes the form—

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$$ 17

4. In a one-dimensional infinite solid, $-\infty < x < \infty$, the surface $a < x < b$ is initially maintained at temperature T_0 and at zero temperature everywhere outside the surface show that :

$$T(xt) = \frac{T_0}{2}\left[erf\left(\frac{b-x}{\sqrt{4\alpha t}}\right) - erf\left(\frac{a-x}{\sqrt{4\alpha t}}\right)\right]$$ 17

Where *erf* is an error function.

5. Obtain the periodic solution of the wave equation in the form—

$$U\,(x,\, t) = Al^{i(kx \pm wt)}$$

Where $i = \sqrt{-1}$, $k = \pm\frac{w}{c}$, A is constant, and hence define various terms involved in wave propagation. 17